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# Wave

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In physics, a **wave** is an oscillation accompanied by a transfer of energy that travels through a medium (space or mass). Frequency refers to the addition of time. *Wave motion* transfers energy from one point to another, which displace particles of the transmission medium—that is, with little or no associated mass transport. Waves consist, instead, of oscillations or vibrations (of a physical quantity), around almost fixed locations.

There are two main types of waves. Mechanical waves propagate through a medium, and the substance of this medium is deformed. Restoring forces then reverse the deformation. For example, sound waves propagate via air molecules colliding with their neighbors. When the molecules collide, they also bounce away from each other (a restoring force). This keeps the molecules from continuing to travel in the direction of the wave.

The second main type, electromagnetic waves, do not require a medium. Instead, they consist of periodic oscillations of electrical and magnetic fields originally generated by charged particles, and can therefore travel through a vacuum. These types vary in wavelength, and include radio waves, microwaves, infrared radiation, visible light, ultraviolet radiation, X-rays and gamma rays.

Waves are described by a wave equation which sets out how the disturbance proceeds over time. The mathematical form of this equation varies depending on the type of wave. Further, the behavior of particles in quantum mechanics are described by waves. In addition, gravitational waves also travel through space, which are a result of a vibration or movement in gravitational fields.

A wave can be transverse, where a disturbance creates oscillations that are perpendicular to the propagation of energy transfer, or longitudinal: the oscillations are parallel to the direction of energy propagation. While mechanical waves can be both transverse and longitudinal, all electromagnetic waves are transverse in free space.

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# General features

A single, all-encompassing definition for the term *wave* is not straightforward. A vibration can be defined as a *back-and-forth* motion around a reference value. However, a vibration is not necessarily a wave. An attempt to define the necessary and sufficient characteristics that qualify a phenomenon to be called a *wave* results in a fuzzy border line.

The term *wave* is often intuitively understood as referring to a transport of spatial disturbances that are generally not accompanied by a motion of the medium occupying this space as a whole. In a wave, the energy of a vibration is moving away from the source in the form of a disturbance within the surrounding medium (Hall 1980, p. 8). However, this motion is problematic for a standing wave (for example, a wave on a string), where energy is moving in both directions equally,



Surface waves in water showing water ripples

or for electromagnetic (e.g., light) waves in a vacuum, where the concept of medium does not apply and interaction with a target is the key to wave detection and practical applications. There are water waves on the ocean surface; gamma waves and light waves emitted by the Sun; microwaves used in microwave ovens and in radar equipment; radio waves broadcast by radio stations; and sound waves generated by radio receivers, telephone handsets and living creatures (as voices), to mention only a few wave phenomena.

It may appear that the description of waves is closely related to their physical origin for each specific instance of a wave process. For example, acoustics is distinguished from optics in that sound waves are related to a mechanical rather than an electromagnetic wave transfer caused by vibration. Concepts such as mass, momentum, inertia, or elasticity, become therefore crucial in describing acoustic (as distinct from optic) wave processes. This difference in origin introduces certain wave characteristics particular to the properties of the medium involved. For example, in the case of air: vortices, radiation pressure, shock waves etc.; in the case of solids: Rayleigh waves, dispersion; and so on....

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Other properties, however, although usually described in terms of origin, may be generalized to all waves. For such reasons, wave theory represents a particular branch of physics that is concerned with the properties of wave processes independently of their physical origin. For example, based on the mechanical origin of acoustic waves, a moving disturbance in space—time can exist if and only if the medium involved is neither infinitely stiff nor infinitely pliable. If all the parts making up a medium were rigidly *bound*, then they would all vibrate as one, with no delay in the transmission of the vibration and therefore no wave motion. On the other hand, if all the parts were independent, then there would not be any transmission of the vibration and again, no wave motion. Although the above statements are meaningless in the case of waves that do not require a medium, they reveal a characteristic that is relevant to all waves regardless of origin: within a wave, the phase of a vibration (that is, its position within the vibration cycle) is different for adjacent points in space because the vibration reaches these points at different times.

# Mathematical description of one-dimensional waves

# Wave equation

Consider a traveling transverse wave (which may be a pulse) on a string (the medium). Consider the string to have a single spatial dimension. Consider this wave as traveling

- in the x direction in space. E.g., let the positive x direction be to the right, and the negative x direction be to the left.
- with constant amplitude u
- with constant velocity v, where v is
  - independent of wavelength (no dispersion)
  - independent of amplitude (linear media, not nonlinear). [2]
- with constant waveform, or shape

This wave can then be described by the two-dimensional functions

$$u(x,t) = F(x - v t)$$
 (waveform  $F$  traveling to the right)  
 $u(x,t) = G(x + v t)$  (waveform  $G$  traveling to the left)

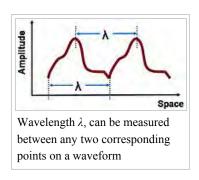
or, more generally, by d'Alembert's formula: [3]

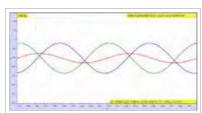
$$u(x,t) = F(x - vt) + G(x + vt).$$

representing two component waveforms F and G traveling through the medium in opposite directions. A generalized representation of this wave can be obtained<sup>[4]</sup> as the partial differential equation

$$rac{1}{v^2}rac{\partial^2 u}{\partial t^2}=rac{\partial^2 u}{\partial x^2}.$$

General solutions are based upon Duhamel's principle.<sup>[5]</sup>





animation for 2 wavelength, green wave traverse to the right while blue wave transverse left, the net red wave amplitude at each point is the sum of the amplitudes of the individual waves note that f(x,t) + g(x,t) = u(x,t)

#### Wave forms

The form or shape of F in d'Alembert's formula involves the argument x - vt. Constant values of this argument correspond to constant values of F, and these constant values occur if x increases at the same rate that vt increases. That is, the wave shaped like the function F will move in the positive x-direction at velocity v (and G will propagate at the same speed in the negative x-direction). [6]

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In the case of a periodic function F with period  $\lambda$ , that is,  $F(x + \lambda - vt) = F(x - vt)$ , the periodicity of F in space means that a snapshot of the wave at a given time t finds the wave varying periodically in space with period  $\lambda$  (the wavelength of the wave). In a similar fashion, this periodicity of F implies a periodicity in time as well: F(x - v(t + T)) = F(x - vt) provided  $vT = \lambda$ , so an observation of the wave at a fixed location x finds the wave undulating periodically in time with period  $T = \lambda/v$ . [7]

# Amplitude and modulation

The amplitude of a wave may be constant (in which case the wave is a *c.w.* or *continuous wave*), or may be *modulated* so as to vary with time and/or position. The outline of the variation in amplitude is called the *envelope* of the wave. Mathematically, the modulated wave can be written in the form:<sup>[8][9][10]</sup>

$$u(x,t) = A(x,t)\sin(kx - \omega t + \phi) \ ,$$

where A(x, t) is the amplitude envelope of the wave, k is the *wavenumber* and  $\phi$  is the *phase*. If the group velocity  $v_g$  (see below) is wavelength-independent, this equation can be simplified as:<sup>[11]</sup>

$$u(x,t) = A(x - v_q t) \sin(kx - \omega t + \phi) ,$$

showing that the envelope moves with the group velocity and retains its shape. Otherwise, in cases where the group velocity varies with wavelength, the pulse shape changes in a manner often described using an *envelope equation*.<sup>[11][12]</sup>

# Phase velocity and group velocity

There are two velocities that are associated with waves, the phase velocity and the group velocity. To understand them, one must consider several types of waveform. For simplification, examination is restricted to one dimension.

The most basic wave (a form of plane wave) may be expressed in the form:

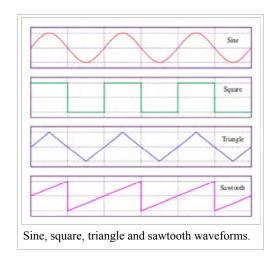
$$\psi(x,t) = Ae^{i(kx-\omega t)}$$
,

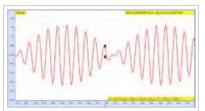
which can be related to the usual sine and cosine forms using Euler's formula. Rewriting the argument,  $kx-\omega t=\left(\frac{2\pi}{\lambda}\right)(x-vt)$ , makes clear that this expression describes a vibration of wavelength  $\lambda=\frac{2\pi}{k}$  traveling in the x-direction with a constant *phase velocity*  $v_p=\frac{\omega}{k}$ . [13]

The other type of wave to be considered is one with localized structure described by an envelope, which may be expressed mathematically as, for example:

$$\psi(x,t) = \int_{-\infty}^{\infty} \, dk_1 \; A(k_1) \; e^{i(k_1 x - \omega t)} \; ,$$

where now  $A(k_1)$  (the integral is the inverse Fourier transform of A(k1)) is a function exhibiting a sharp peak in a region of wave vectors  $\Delta k$  surrounding the point  $k_1 = k$ . In exponential form:





amplitude modulation can be achieved through  $f(x,t) = 1.00*\sin(2*pi/0.10*(x-1.00*t))$  and  $g(x,t) = 1.00*\sin(2*pi/0.11*(x-1.00*t))$  only the resultant is visible to improve clarity of waveform

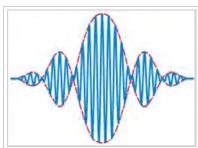


Illustration of the *envelope* (the slowly varying red curve) of an amplitude-modulated wave. The fast varying blue curve is the *carrier* wave, which is being modulated.

# M--M--M--M

Frequency dispersion in groups of gravity waves on the surface of deep water. The red dot moves with the phase velocity, and the green dots propagate with the group velocity.

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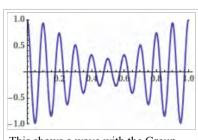
$$A=A_{o}(k_1)e^{ilpha(k_1)}$$
 ,

with  $A_0$  the magnitude of A. For example, a common choice for  $A_0$  is a Gaussian wave packet:<sup>[14]</sup>

$$A_o(k_1) = N \ e^{-\sigma^2(k_1-k)^2/2} \ ,$$

where  $\sigma$  determines the spread of  $k_1$ -values about k, and N is the amplitude of the wave.

The exponential function inside the integral for  $\psi$  oscillates rapidly with its argument, say  $\varphi(k_1)$ , and where it varies rapidly, the exponentials cancel each other



This shows a wave with the Group velocity and Phase velocity going in different directions.

out, interfere destructively, contributing little to  $\psi$ .<sup>[13]</sup> However, an exception occurs at the location where the argument  $\varphi$  of the exponential varies slowly. (This observation is the basis for the method of stationary phase for evaluation of such integrals.<sup>[15]</sup>) The condition for  $\varphi$  to vary slowly is that its rate of change with  $k_1$  be small; this rate of variation is:<sup>[13]</sup>

$$\left. rac{darphi}{dk_1} 
ight|_{k_1=k} = x - t rac{d\omega}{dk_1} 
ight|_{k_1=k} + \left. rac{dlpha}{dk_1} 
ight|_{k_1=k} \, ,$$

where the evaluation is made at  $k_1 = k$  because  $A(k_1)$  is centered there. This result shows that the position x where the phase changes slowly, the position where  $\psi$  is appreciable, moves with time at a speed called the *group velocity*:

$$v_g = rac{d\omega}{dk} \ .$$

The group velocity therefore depends upon the dispersion relation connecting  $\omega$  and k. For example, in quantum mechanics the energy of a particle represented as a wave packet is  $E = \hbar \omega = (\hbar k)^2/(2m)$ . Consequently, for that wave situation, the group velocity is

$$v_g = rac{\hbar k}{m} \, ,$$

showing that the velocity of a localized particle in quantum mechanics is its group velocity. Because the group velocity varies with k, the shape of the wave packet broadens with time, and the particle becomes less localized. In other words, the velocity of the constituent waves of the wave packet travel at a rate that varies with their wavelength, so some move faster than others, and they cannot maintain the same interference pattern as the wave propagates.

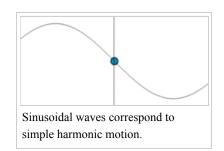
# Sinusoidal waves

Mathematically, the most basic wave is the (spatially) one-dimensional sine wave (or *harmonic wave* or *sinusoid*) with an amplitude u described by the equation:

$$u(x,t) = A\sin(kx - \omega t + \phi)$$
,

where

- A is the maximum amplitude of the wave, maximum distance from the highest point of the disturbance in the medium (the crest) to the equilibrium point during one wave cycle. In the illustration to the right, this is the maximum vertical distance between the baseline and the wave.
- x is the space coordinate
- t is the time coordinate
- *k* is the wavenumber



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- $\omega$  is the angular frequency
- $\phi$  is the phase constant.

The units of the amplitude depend on the type of wave. Transverse mechanical waves (e.g., a wave on a string) have an amplitude expressed as a distance (e.g., meters), longitudinal mechanical waves (e.g., sound waves) use units of pressure (e.g., pascals), and electromagnetic waves (a form of transverse vacuum wave) express the amplitude in terms of its electric field (e.g., volts/meter).

The wavelength  $\lambda$  is the distance between two sequential crests or troughs (or other equivalent points), generally is measured in meters. A wavenumber k, the spatial frequency of the wave in radians per unit distance (typically per meter), can be associated with the wavelength by the relation

$$k = \frac{2\pi}{\lambda}.$$

The period T is the time for one complete cycle of an oscillation of a wave. The frequency f is the number of periods per unit time (per second) and is typically measured in hertz. These are related by:

$$f = \frac{1}{T}.$$

In other words, the frequency and period of a wave are reciprocals.

The angular frequency  $\omega$  represents the frequency in radians per second. It is related to the frequency or period by

$$\omega=2\pi f=rac{2\pi}{T}.$$

The wavelength  $\lambda$  of a sinusoidal waveform traveling at constant speed v is given by: [17]

$$\lambda = rac{v}{f},$$

where v is called the phase speed (magnitude of the phase velocity) of the wave and f is the wave's frequency.

Wavelength can be a useful concept even if the wave is not periodic in space. For example, in an ocean wave approaching shore, the incoming wave undulates with a varying *local* wavelength that depends in part on the depth of the sea floor compared to the wave height. The analysis of the wave can be based upon comparison of the local wavelength with the local water depth.<sup>[18]</sup>

Although arbitrary wave shapes will propagate unchanged in lossless linear time-invariant systems, in the presence of dispersion the sine wave is the unique shape that will propagate unchanged but for phase and amplitude, making it easy to analyze. <sup>[19]</sup> Due to the Kramers–Kronig relations, a linear medium with dispersion also exhibits loss, so the sine wave propagating in a dispersive medium is attenuated in certain frequency ranges that depend upon the medium. <sup>[20]</sup> The sine function is periodic, so the sine wave or sinusoid has a wavelength in space and a period in time. <sup>[21][22]</sup>

The sinusoid is defined for all times and distances, whereas in physical situations we usually deal with waves that exist for a limited span in space and duration in time. Fortunately, an arbitrary wave shape can be decomposed into an infinite set of sinusoidal waves by the use of Fourier analysis. As a result, the simple case of a single sinusoidal wave can be applied to more general cases. [23][24] In particular, many media are linear, or nearly so, so the calculation of arbitrary wave behavior can be found by adding up responses to individual sinusoidal waves using the superposition principle to find the solution for a general waveform. [25] When a medium is nonlinear, the response to complex waves cannot be determined from a sine-wave decomposition.

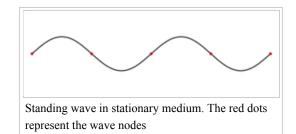
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## Plane waves

# **Standing waves**

A standing wave, also known as a *stationary wave*, is a wave that remains in a constant position. This phenomenon can occur because the medium is moving in the opposite direction to the wave, or it can arise in a stationary medium as a result of interference between two waves traveling in opposite directions.

The *sum* of two counter-propagating waves (of equal amplitude and frequency) creates a *standing wave*. Standing waves commonly arise when a boundary blocks further propagation of the wave, thus causing wave reflection, and therefore introducing a counter-propagating wave.



For example, when a violin string is displaced, transverse waves propagate out to where the string is held in place at the bridge and the nut, where the waves are reflected back. At the bridge and nut, the two opposed waves are in antiphase and cancel each other, producing a node. Halfway between two nodes there is an antinode, where the two counter-propagating waves *enhance* each other maximally. There is no net propagation of energy over time.







One-dimensional standing waves; the fundamental mode and the first 5 overtones.

A two-dimensional standing wave on a disk; this is the fundamental mode.

A standing wave on a disk with two nodal lines crossing at the center; this is an overtone.

# Physical properties

Waves exhibit common behaviors under a number of standard situations, e. g.

#### Transmission and media

Waves normally move in a straight line (i.e. rectilinearly) through a *transmission medium*. Such media can be classified into one or more of the following categories:

- A bounded medium if it is finite in extent, otherwise an unbounded medium
- A linear medium if the amplitudes of different waves at any particular point in the medium can be added
- A uniform medium or homogeneous medium if its physical properties are unchanged at different locations in space
- An anisotropic medium if one or more of its physical properties differ in one or more directions
- An *isotropic medium* if its physical properties are the *same* in all directions



Light beam exhibiting reflection, refraction, transmission and dispersion when encountering a prism

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## **Absorption**

Absorption of waves means, if a kind of wave strikes a matter, it will be absorbed by the matter. When a wave with that same natural frequency impinges upon an atom, then the electrons of that atom will be set into vibrational motion. If a wave of a given frequency strikes a material with electrons having the same vibrational frequencies, then those electrons will absorb the energy of the wave and transform it into vibrational motion.

## Reflection

When a wave strikes a reflective surface, it changes direction, such that the angle made by the incident wave and line normal to the surface equals the angle made by the reflected wave and the same normal line.

#### Interference

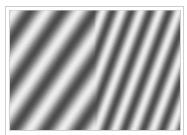
Waves that encounter each other combine through superposition to create a new wave called an interference pattern. Important interference patterns occur for waves that are in phase.

#### Refraction

Refraction is the phenomenon of a wave changing its speed. Mathematically, this means that the size of the phase velocity changes. Typically, refraction occurs when a wave passes from one medium into another. The amount by which a wave is refracted by a material is given by the refractive index of the material. The directions of incidence and refraction are related to the refractive indices of the two materials by Snell's law.

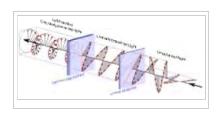
#### **Diffraction**

A wave exhibits diffraction when it encounters an obstacle that bends the wave or when it spreads after emerging from an opening. Diffraction effects are more pronounced when the size of the obstacle or opening is comparable to the wavelength of the wave.



Sinusoidal traveling plane wave entering a region of lower wave velocity at an angle, illustrating the decrease in wavelength and change of direction (refraction) that results

#### **Polarization**



The phenomenon of polarization arises when wave motion can occur simultaneously in two orthogonal directions. Transverse waves can be polarized, for instance. When polarization is used as a descriptor without qualification, it usually refers to the special, simple case of linear polarization. A transverse wave is linearly polarized if it oscillates in only one direction or plane. In the case of linear polarization, it is often useful to add the relative orientation of that plane, perpendicular to the direction of travel, in which the oscillation occurs, such as "horizontal" for instance, if the plane of polarization is parallel to the ground.

Electromagnetic waves propagating in free space, for instance, are transverse; they can be polarized by the use of a polarizing filter.

Longitudinal waves, such as sound waves, do not exhibit polarization. For these waves there is only one direction of oscillation, that is, along the direction of travel.

#### **Dispersion**

A wave undergoes dispersion when either the phase velocity or the group velocity depends on the wave frequency. Dispersion is most easily seen by letting white light pass through a prism, the result of which is to produce the spectrum of colours of the rainbow. Isaac Newton performed experiments with light and prisms, presenting his findings in the *Opticks* (1704) that white light consists of several colours and that these colours cannot be decomposed any further.<sup>[26]</sup>

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# **Mechanical** waves

# Waves on strings

The speed of a transverse wave traveling along a vibrating string (v) is directly proportional to the square root of the tension of the string (T) over the linear mass density ( $\mu$ ):

$$v=\sqrt{rac{T}{\mu}},$$

where the linear density  $\mu$  is the mass per unit length of the string.

# Schematic of light being dispersed by a prism. Click to see animation.

#### **Acoustic waves**

Acoustic or sound waves travel at speed given by

$$v=\sqrt{rac{B}{
ho_0}},$$

or the square root of the adiabatic bulk modulus divided by the ambient fluid density (see speed of sound).

#### Water waves

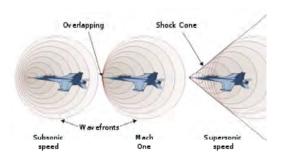
- Ripples on the surface of a pond are actually a combination of transverse and longitudinal waves; therefore, the points on the surface follow orbital paths.
- wave phase : t / T = 3.000
- Sound—a mechanical wave that propagates through gases, liquids, solids and plasmas;
- Inertial waves, which occur in rotating fluids and are restored by the Coriolis effect;
- Ocean surface waves, which are perturbations that propagate through water.

#### Seismic waves

#### **Shock waves**

#### Other

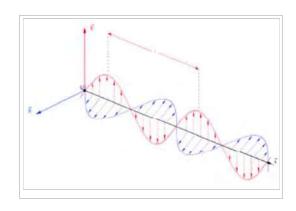
- Waves of traffic, that is, propagation of different densities of motor vehicles, and so forth, which can be modeled as kinematic waves<sup>[27]</sup>
- Metachronal wave refers to the appearance of a traveling wave produced by coordinated sequential actions.
- It is worth noting that the mass-energy equivalence equation can be solved for this form:  $c = \sqrt{\frac{e}{m}}$ .



# **Electromagnetic waves**

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An electromagnetic wave consists of two waves that are oscillations of the electric and magnetic fields. An electromagnetic wave travels in a direction that is at right angles to the oscillation direction of both fields. In the 19th century, James Clerk Maxwell showed that, in vacuum, the electric and magnetic fields satisfy the wave equation both with speed equal to that of the speed of light. From this emerged the idea that light is an electromagnetic wave. Electromagnetic waves can have different frequencies (and thus wavelengths), giving rise to various types of radiation such as radio waves, microwaves, infrared, visible light, ultraviolet, X-rays, and Gamma rays.



# **Quantum mechanical waves**

# Schrödinger equation

The Schrödinger equation describes the wave-like behavior of particles in quantum mechanics. Solutions of this equation are wave functions which can be used to describe the probability density of a particle.

# **Dirac equation**

The Dirac equation is a relativistic wave equation detailing electromagnetic interactions. Dirac waves accounted for the fine details of the hydrogen spectrum in a completely rigorous way. The wave equation also implied the existence of a new form of matter, antimatter, previously unsuspected and unobserved and which was experimentally confirmed. In the context of quantum field theory, the Dirac equation is reinterpreted to describe quantum fields corresponding to spin-½ particles.

# de Broglie waves

Louis de Broglie postulated that all particles with momentum have a wavelength

$$\lambda = \frac{h}{p},$$

where h is Planck's constant, and p is the magnitude of the momentum of the particle. This hypothesis was at the basis of quantum mechanics. Nowadays, this wavelength is called the de Broglie wavelength. For example, the electrons in a CRT display have a de Broglie wavelength of about  $10^{-13}$  m.

A wave representing such a particle traveling in the *k*-direction is expressed by the wave function as follows:

$$\psi({f r},\ t=0)=A\ e^{i{f k}\cdot{f r}}\ ,$$

where the wavelength is determined by the wave vector  $\mathbf{k}$  as:

$$\lambda = \frac{2\pi}{h}$$
,

and the momentum by:

$$\mathbf{p} = \hbar \mathbf{k}$$
.



A propagating wave packet; in general, the *envelope* of the wave packet moves at a different speed than the constituent waves.<sup>[28]</sup>

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However, a wave like this with definite wavelength is not localized in space, and so cannot represent a particle localized in space. To localize a particle, de Broglie proposed a superposition of different wavelengths ranging around a central value in a wave packet, [29] a waveform often used in quantum mechanics to describe the wave function of a particle. In a wave packet, the wavelength of the particle is not precise, and the local wavelength deviates on either side of the main wavelength value.

In representing the wave function of a localized particle, the wave packet is often taken to have a Gaussian shape and is called a *Gaussian wave packet*.<sup>[30]</sup> Gaussian wave packets also are used to analyze water waves.<sup>[31]</sup>

For example, a Gaussian wavefunction  $\psi$  might take the form: [32]

$$\psi(x,\ t=0)=A\ \exp\Bigl(-rac{x^2}{2\sigma^2}+ik_0x\Bigr)\ ,$$

at some initial time t = 0, where the central wavelength is related to the central wave vector  $k_0$  as  $\lambda_0 = 2\pi / k_0$ . It is well known from the theory of Fourier analysis, [33] or from the Heisenberg uncertainty principle (in the case of quantum mechanics) that a narrow range of wavelengths is necessary to produce a localized wave packet, and the more localized the envelope, the larger the spread in required wavelengths. The Fourier transform of a Gaussian is itself a Gaussian. [34] Given the Gaussian:

$$f(x)=e^{-x^2/(2\sigma^2)}\;,$$

the Fourier transform is:

$$ilde{f}\left(k
ight)=\sigma e^{-\sigma^2k^2/2}\;.$$

The Gaussian in space therefore is made up of waves:

$$f(x) = rac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} \, ilde{f}(k) e^{ikx} \; dk \, ;$$

that is, a number of waves of wavelengths  $\lambda$  such that  $k\lambda = 2 \pi$ .

The parameter  $\sigma$  decides the spatial spread of the Gaussian along the *x*-axis, while the Fourier transform shows a spread in wave vector *k* determined by  $1/\sigma$ . That is, the smaller the extent in space, the larger the extent in *k*, and hence in  $\lambda = 2\pi/k$ .

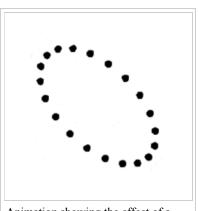
# **Gravity waves**

Gravity waves are waves generated in a fluid medium or at the interface between two media when the force of gravity or buoyancy tries to restore equilibrium. A ripple on a pond is one example.

## **Gravitational waves**

Gravitational waves also travel through space. The first observation of gravitational waves was announced on 11 February 2016.<sup>[35]</sup> Gravitational waves are disturbances in the curvature of spacetime, predicted by Einstein's theory of general relativity.

# WKB method



Animation showing the effect of a cross-polarized gravitational wave on a ring of test particles

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In a nonuniform medium, in which the wavenumber k can depend on the location as well as the frequency, the phase term kx is typically replaced by the integral of k(x)dx, according to the WKB method. Such nonuniform traveling waves are common in many physical problems, including the mechanics of the cochlea and waves on hanging ropes.

# See also

Index of wave articles

# Waves in general

- Wave equation, general
- Wave propagation, any of the ways in which waves travel
- Interference (wave propagation), a phenomenon in which two waves superpose to form a resultant wave
- Mechanical wave, in media transmission
- Wave Motion (journal), a scientific journal
- Wavefront, an advancing surface of wave propagation

#### **Parameters**

- Phase (waves), offset or angle of a sinusoidal wave function at its origin
- Standing wave ratio, in telecommunications
- Wavelength
- Wavenumber
- Wave period

#### Waveforms

- Creeping wave, a wave diffracted around a sphere
- Evanescent wave
- Longitudinal wave
- Periodic travelling wave

- Sine wave
- Square wave
- Standing wave
- Transverse wave

#### **Electromagnetic waves**

- Electromagnetic wave
- Electromagnetic wave equation, describes electromagnetic wave propagation
- Earth-Ionosphere waveguide, in radio transmission
- Microwave, a form of electromagnetic radiation

#### In fluids

- Airy wave theory, in fluid dynamics
- Capillary wave, in fluid dynamics
- Cnoidal wave, in fluid dynamics
- Edge wave, a surface gravity wave fixed by refraction against a rigid boundary
- Faraday wave, a type of wave in liquids
- Gravity wave, in fluid dynamics

- Sound wave, a wave of sound through a medium such as air or water
- Shock wave, in aerodynamics
- Internal wave, a wave within a fluid medium
- Tidal wave, an alternative name for a tsunami
- Tollmien–Schlichting wave, in fluid dynamics

#### In quantum mechanics

- Bloch wave
- Matter wave
- Pilot wave, in Bohmian mechanics

- Wave function
- Wave packet
- Wave-particle duality

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# In relativity

- Gravitational wave, in relativity theory
- Relativistic wave equations, wave equations that consider special relativity
- pp-wave spacetime, a set of exact solutions to Einstein's field equation

## Other specific types of waves

- Alfvén wave, in particle science
- Atmospheric wave, a periodic disturbance in the fields of atmospheric variables
- Fir wave, a forest configuration
- Lamb waves, in solid materials

- Rayleigh waves, surface acoustic waves that travel on solids
- Spin wave, in magnetism
- Spin-density wave, in solid materials
- Trojan wave packet, in particle science
- Waves in plasmas, in particle science

## Related topics

- Beat (acoustics)
- Cymatics
- Doppler effect
- Envelope detector
- Group velocity
- Harmonic
- Index of wave articles
- Inertial wave
- List of waves named after people
- Phase velocity
- Reaction—diffusion system

- Resonance
- Ripple tank
- Rogue wave
- Shallow water equations
- Shive wave machine
- Sound
- Standing wave
- Transmission medium
- Wave turbulence
- Wind wave

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# **External links**

- Interactive Visual Representation of Waves (http://resonanceswavesandfields.blogspot.com/2007/08/true-waves.html)
- Linear and nonlinear waves (http://www.scholarpedia.org/article/Linear\_and\_nonlinear\_waves)
- Science Aid: Wave properties—Concise guide aimed at teens (http://www.scienceaid.co.uk/physics/waves/properties.html)
- Easy JavaScript Simulation Model of One Dimensional Wave Interference (http://iwant2study.org/lookangejss/04waves\_11superposition/ejss\_model\_wave1d01/wave1d01\_Simulation.xhtml)
- Simulation of diffraction of water wave passing through a gap (http://www.phy.hk/wiki/englishhtm/Diffraction.htm)
- Simulation of interference of water waves (http://www.phy.hk/wiki/englishhtm/Interference.htm)
- Simulation of longitudinal traveling wave (http://www.phy.hk/wiki/englishhtm/Lwave.htm)
- Simulation of stationary wave on a string (http://www.phy.hk/wiki/englishhtm/StatWave.htm)
- Simulation of transverse traveling wave (http://www.phy.hk/wiki/englishhtm/TwaveA.htm)
- Sounds Amazing—AS and A-Level learning resource for sound and waves (http://www.acoustics.salford.ac.uk/feschools/)
- chapter from an online textbook (http://www.lightandmatter.com/html books/lm/ch19/ch19.html)
- Simulation of waves on a string (http://www.physics-lab.net/applets/mechanical-waves)
- of longitudinal and transverse mechanical wave (http://www.cbu.edu/~jvarrian/applets/waves1/lontra\_g.htm-simulation)
- MIT OpenCourseWare 8.03: Vibrations and Waves (http://ocw.mit.edu/courses/physics/8-03-physics-iii-vibrations-and-waves-fall-2004/) Free, independent study course with video lectures, assignments, lecture notes and exams.

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