Simple beam formulae

or do your own stressing

The basic formula for the state of affairs at any given point along a beam is:

$$\frac{M}{I} = \frac{s}{y} = \frac{E}{R}$$

Where $M =$ the *bending moment* to which the beam is subjected at this point. Bending moment is load multiplied by distance, e.g. in inch pounds. If M is not, for a given case, obvious from first principles it may be obtained from the data given below.

$I =$ the *moment of inertia* of the section of the beam at this point, e.g. in inches^4. Moments of inertia are treated in text-books of elementary mechanics. The moment of inertia in question here is that about the neutral axis which, in most beam problems, will be a line through the centre of gravity of the section. If the I of the section is not already known it is usually easily calculated by applying one of the following formulae.

For a rectangle:

$$I = \frac{bd^3}{12}$$

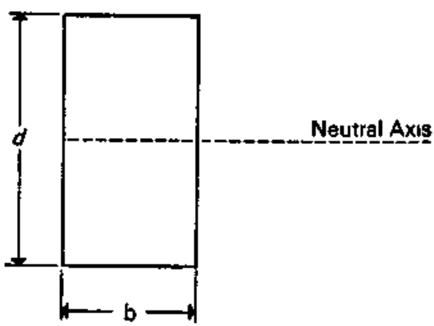

Figure 1.

Hence sections made up of rectangles, such as rectangular tubes, *H* beams and channels can often be calculated by subtracting the I of the empty areas.

For a circle:

$$I = \frac{\pi r^4}{4}$$

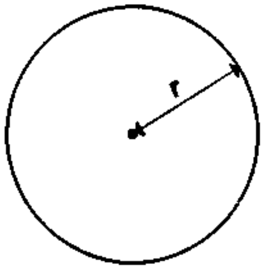

Figure 2.

Again hollow tubes can be calculated by subtraction.

s = stress in material.

y = distance from the neutral axis. In a symmetrical section this is the distance from the mid-point or centre of gravity of the section.

E = Young's modulus.

R = radius of curvature of beam when it bends under load.

Of these various properties we most often want to calculate the stress in the beam and so:

$$s = \frac{My}{I}$$

For a simple rectangular beam:

$$I = \frac{bd^3}{12} \quad \text{and} \quad y = \frac{d}{2}$$

at the surface where the stress is greatest. So:

$$s_{max} = 6\frac{M}{bd^2}$$

Which is why a beam twice as thick is four times as strong and so on.

For specific beams the following information is useful. Information covering virtually every imaginable form of beam is given in *Formulas for Stress and Strain* by R. J. Roark (McGraw-Hill, 1954) which should be referred to for more complicated cases.

Point loads

(*a*) *Simple cantilever length l with point load W at end.*

FOR BENDING MOMENT

at any point distant x from the end:

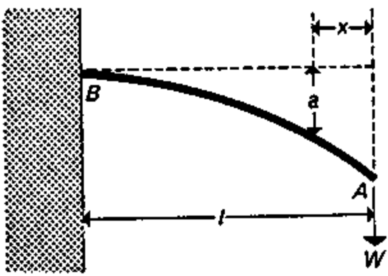

Figure 3.

$$M = Wx$$

$$\text{Max } M = Wl \text{ at } B$$

FOR DEFLECTION

at any point:

$$a = \frac{1}{6}\frac{W}{EI}(x^3 - 3l^2x + 2l^3)$$

$$\text{Max } a = \frac{1}{3}\frac{Wl^3}{EI} \text{ at } A$$

$$\text{Slope at } A = \frac{1}{2}\frac{Wl^2}{EI} \text{ radians}$$

(b) *Simply supported beam length l with point load W in middle.*

FOR BENDING MOMENT

at any point:

$$M = \frac{1}{2} Wx$$

$$\text{Max } M = \frac{Wl}{4}$$

FOR DEFLECTION

at any point:

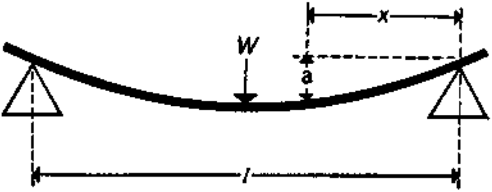

Figure 4.

$$a = \frac{1}{48} \frac{W}{EI}(3l^2x - 4x^3)$$

$$\text{Max } a = \frac{1}{48} \frac{Wl^3}{EI} \text{ at centre}$$

$$\text{Slope at ends} = \frac{1}{16} \frac{Wl^2}{EI} \text{ radians}$$

(So, *inter alia*, a beam twice as long deflects eight times as much.)

NOTE ON CONVERSION OF UNITS

Length	1 metre	= 32·8 feet
	1 foot	= 30·48 centimetres (exactly)
	1 micron (μm)	= 10^{-4} centimetre = $3{\cdot}937 \times 10^{-5}$ in.
	1 nanometre	= 10 Ångströms = 10^{-9} metre
Area	1 square inch	= 6·4516 square centimetres exactly
Force	1 kg force	= 2·205 lb. force
		= 9·81 Newtons
		= $0{\cdot}98 \times 10^6$ dynes
Stress or pressure		
	1 p.s.i.	= 6895 Newtons per square metre
		= 0·0705 kg per square centimetre
		= $6{\cdot}9 \times 10^4$ dynes per square centimetre
	1 atmosphere	= 14·696 p.s.i. 1 MN/m² = 10^6N/m² = 146 p.s.i.
		= 1·033 kg per square centimetre
Energy	1 Joule	= 10^7 ergs
		= 0·239 calories
		= 0·734 ft. lb.