Order of magnitude

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Orders of magnitude are written in powers of 10. For example, the order of magnitude of 1500 is 3, since 1500 may be written as 1.5×10^3.

Differences in order of magnitude can be measured on a base-10 logarithmic scale in “decades” (i.e., factors of ten).[1] Examples of numbers of different magnitudes can be found at Orders of magnitude (numbers).

> We say two numbers have the same order of magnitude of a number if the big one divided by the little one is less than 10. For example, 23 and 82 have the same order of magnitude, but 23 and 820 do not.
>
> — John C. Baez[2]

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Definition

Generally, the order of magnitude of a number is the smallest power of 10 required to represent that number.[3] To work out the order of magnitude of a number N, the number is first expressed in the following form:

$$N = a \times 10^b,$$

where $0.5 < a \leq 5$. Then, b represents the order of magnitude of the number. The order of magnitude can be a positive integer, zero, or a negative integer. The table below enumerates the order of magnitude of some numbers in light of this definition:

Number N	**Expression in $N = a \times 10^b$**	**Order of magnitude b**
0.325	3.25×10^{-1}	–1
0.5	5×10^{-1}	–1
5	5×10^{0}	0
7	0.7×10^{1}	1
44	4.4×10^{1}	1

Uses

Orders of magnitude are used to make approximate comparisons. If numbers differ by one order of magnitude, *x* is *about* ten times different in quantity than *y*. If values differ by two orders of magnitude, they differ by a factor of about 100. Two numbers of the same order of magnitude have roughly the same scale: the larger value is less than ten times the smaller value.

In words (long scale)	In words (short scale)	Prefix (Symbol)	Decimal	Power of ten	Order of magnitude
quadrillionth	septillionth	yocto- (y)	0.000 000 000 000 000 000 000 001	10^{-24}	−24
trilliardth	sextillionth	zepto- (z)	0.000 000 000 000 000 000 001	10^{-21}	−21
trillionth	quintillionth	atto- (a)	0.000 000 000 000 000 001	10^{-18}	−18
billiardth	quadrillionth	femto- (f)	0.000 000 000 000 001	10^{-15}	−15
billionth	trillionth	pico- (p)	0.000 000 000 001	10^{-12}	−12
milliardth	billionth	nano- (n)	0.000 000 001	10^{-9}	−9
millionth	millionth	micro- (μ)	0.000 001	10^{-6}	−6
thousandth	thousandth	milli- (m)	0.001	10^{-3}	−3
hundredth	hundredth	centi- (c)	0.01	10^{-2}	−2
tenth	tenth	deci- (d)	0.1	10^{-1}	−1
one	one	–	1	10^{0}	0
ten	ten	deca- (da)	10	10^{1}	1
hundred	hundred	hecto- (h)	100	10^{2}	2
thousand	thousand	kilo- (k)	1000	10^{3}	3
million	million	mega- (M)	1 000 000	10^{6}	6
milliard	billion	giga- (G)	1 000 000 000	10^{9}	9
billion	trillion	tera- (T)	1 000 000 000 000	10^{12}	12
billiard	quadrillion	peta- (P)	1 000 000 000 000 000	10^{15}	15
trillion	quintillion	exa- (E)	1 000 000 000 000 000 000	10^{18}	18
trilliard	sextillion	zetta- (Z)	1 000 000 000 000 000 000 000	10^{21}	21
quadrillion	septillion	yotta- (Y)	1 000 000 000 000 000 000 000 000	10^{24}	24

Calculating the order of magnitude

The order of magnitude of a number is, intuitively speaking, the number of powers of 10 contained in the number. More precisely, the order of magnitude of a number can be defined in terms of the common logarithm, usually as the integer part of the logarithm, obtained by truncation. For example, the number 4 000 000 has a logarithm (in base 10) of 6.602; its order of magnitude is 6. When truncating, a number of this order of magnitude is between 10^6 and 10^7. In a similar example, with the phrase "He had a seven-figure income", the order of magnitude is the number of figures minus one, so it is very easily determined without a calculator to 6. An order of magnitude is an approximate position on a logarithmic scale.

Order-of-magnitude estimate

An order-of-magnitude estimate of a variable whose precise value is unknown is an estimate rounded to the nearest power of ten. For example, an order-of-magnitude estimate for a variable between about 3 billion and 30 billion (such as the human population of the Earth) is 10 billion. To round a number to its nearest order of magnitude, one rounds its logarithm to the nearest integer. Thus 4 000 000, which has a logarithm (in base 10) of 6.602, has 7 as its nearest order of magnitude, because "nearest" implies rounding rather than truncation. For a number written in scientific notation, this logarithmic rounding scale requires rounding up to the next power of ten when the multiplier is greater than the square root of ten (about 3.162). For example, the nearest order of magnitude for 1.7×10^8 is 8, whereas the nearest order of magnitude for 3.7×10^8 is 9. An order-of-magnitude estimate is sometimes also called a zeroth order approximation.

Order of magnitude difference

An order-of-magnitude difference between two values is a factor of 10. For example, the mass of the planet Saturn is 95 times that of Earth, so Saturn is *two orders of magnitude* more massive than Earth. Order-of-magnitude differences are called **decades** when measured on a logarithmic scale.

Non-decimal orders of magnitude

Other orders of magnitude may be calculated using bases other than 10. The ancient Greeks ranked the nighttime brightness of celestial bodies by 6 levels in which each level was the fifth root of one hundred (about 2.512) as bright as the nearest weaker level of brightness, and thus the brightest level being 5 orders of magnitude brighter than the weakest indicates that it is $(100^{1/5})^5$ or a **factor** of 100 times brighter.

The different decimal numeral systems of the world use a larger base to better envision the size of the number, and have created names for the powers of this larger base. The table shows what number the order of magnitude aim at for base 10 and for base 1 000 000. It can be seen that the order of magnitude is included in the number name in this example, because bi- means 2 and tri- means 3 (these make sense in the long scale only), and the suffix -illion tells that the base is 1 000 000. But the number names billion, trillion themselves (here with other meaning than in the first chapter) are not names of the *orders of* magnitudes, they are names of "magnitudes", that is the *numbers* 1 000 000 000 000 etc.

Order of magnitude	Is $\log_{10}$ of	Is $\log_{1\,000\,000}$ of	Short scale	Long scale
1	10	1 000 000	million	million
2	100	1 000 000 000 000	trillion	billion
3	1000	1 000 000 000 000 000 000	quintillion	trillion

SI units in the table at right are used together with SI prefixes, which were devised with mainly base 1000 magnitudes in mind. The IEC standard prefixes with base 1024 were invented for use in electronic technology.

The ancient apparent magnitudes for the brightness of stars uses the base $\sqrt[5]{100} \approx 2.512$ and is reversed. The modernized version has however turned into a logarithmic scale with non-integer values.

Extremely large numbers

For extremely large numbers, a generalized order of magnitude can be based on their double logarithm or super-logarithm. Rounding these downward to an integer gives categories between very "round numbers", rounding them to the nearest integer and applying the inverse function gives the "nearest" round number.

The double logarithm yields the categories:

..., 1.0023–1.023, 1.023–1.26, 1.26–10, 10–10^{10}, 10^{10}–10^{100}, 10^{100}–10^{1000}, ...

(the first two mentioned, and the extension to the left, may not be very useful, they merely demonstrate how the sequence mathematically continues to the left).

The super-logarithm yields the categories:

0–1, 1–10, 10–10^{10}, 10^{10}–$10^{10^{10}}$, $10^{10^{10}}$–$10^{10^{10^{10}}}$, ... or

0–${}^{0}10$, ${}^{0}10$–${}^{1}10$, ${}^{1}10$–${}^{2}10$, ${}^{2}10$–${}^{3}10$, ${}^{3}10$–${}^{4}10$, ...

The "midpoints" which determine which round number is nearer are in the first case:

1.076, 2.071, 1453, 4.20×10^{31}, 1.69×10^{316},...

and, depending on the interpolation method, in the second case

−0.301, 0.5, 3.162, 1453, 1×10^{1453}, $(10\uparrow)^1 10^{1453}$, $(10\uparrow)^2 10^{1453}$,... (see notation of extremely large numbers)

For extremely small numbers (in the sense of close to zero) neither method is suitable directly, but the generalized order of magnitude of the reciprocal can be considered.

Similar to the logarithmic scale one can have a double logarithmic scale (example provided here) and super-logarithmic scale. The intervals above all have the same length on them, with the "midpoints" actually midway. More generally, a point midway between two points corresponds to the generalised f-mean with $f(x)$ the corresponding function log log x or slog x. In the case of log log x, this mean of two numbers (e.g. 2 and 16 giving 4) does not depend on the base of the logarithm, just like in the case of log x (geometric mean, 2 and 8 giving 4), but unlike in the case of log log log x (4 and 65 536 giving 16 if the base is 2, but, otherwise).

See also

- Big O notation
- Decibel
- Names of large numbers
- Names of small numbers
- Number sense
- Orders of approximation
- Orders of magnitude (numbers)

References

1. Brians, Paus. "Orders of Magnitude". Retrieved 9 May 2013.
2. John Baez, 28 November 2012
3. "Order of Magnitude". *Wolfram MathWorld*. Retrieved 3 January 2017. "Physicists and engineers use the phrase "order of magnitude" to refer to the smallest power of ten needed to represent a quantity."

Further reading

- Asimov, Isaac *The Measure of the Universe* (1983)

External links

- The Scale of the Universe 2 (http://htwins.net/scale2/) Interactive tool from Planck length 10^{-35} meters to universe size 10^{27}
- Cosmos – an Illustrated Dimensional Journey from microcosmos to macrocosmos (http://www.shekpvar.net/~dna/Publications/Cosmos/cosmos.html) – from Digital Nature Agency
- Powers of 10 (http://micro.magnet.fsu.edu/primer/java/scienceopticsu/powersof10/index.html), a graphic animated illustration that starts with a view of the Milky Way at 10^{23} meters and ends with subatomic particles at 10^{-16} meters.
- What is Order of Magnitude? (http://www.vendian.org/envelope/TemporaryURL/what_is_oom.html)

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